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MULTIPLICATION THEOREMS FOR $G(N, p, \alpha)$ SUMMABILITY:

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1.1 DEFINITIONS AND NOTATIONS:

Let $\{p_n\}$ and $\{q_n\}$ are two sequences then

$$\Delta\epsilon_n = p_n - p_{n-1} = \Delta p_n$$
$$\Delta\varepsilon_n = q_n - q_{n-1} = \Delta q_n$$

Given two sequences $\{\in\}$ and $\{\alpha\}$, the convolution $(\in * \alpha)_n$ define as,

$$(\epsilon * \alpha)_n = \sum_{v=0}^{n} P_{n-v}\alpha_v \qquad (1.1.1)$$

Operation of convolution is commutative and associative, we note that

$$\sum_{v=1}^{n} \epsilon_v = (\epsilon * 1)_n \qquad (1.1.2)$$

$$\Delta(\epsilon * \alpha)_n = (\Delta\epsilon * \alpha)_n = (\epsilon * \Delta\alpha)_n \qquad (1.1.3)$$

$$N_n^{\epsilon,\alpha}(s) = \frac{(\epsilon * \alpha s)_n}{(\epsilon * \alpha)_n} = \frac{1}{(\epsilon * \alpha)_n}\sum_{v=0}^{n} \epsilon_{n-v}\alpha_v s_v \qquad (1.1.4)$$

$$T_n^{\epsilon,\alpha}(s) = \frac{(\Delta\epsilon * \alpha s)_n}{(\Delta\epsilon * \alpha)_n}$$

$$= \frac{(\epsilon * \Delta\alpha s)_n}{(\epsilon * \Delta\alpha)_n}$$

$$= \frac{1}{(\epsilon * \Delta\alpha)_n} \sum_{v=0}^{n} \epsilon_{n-\mathrm{v}}(\alpha_v, s_v - \alpha_{v-1} s_{v-1}). \qquad (1.1.5)$$

Definition 1:

Absolutely summability: A series $\sum_{n=0}^{\infty} a_n$ is said to be absolutely summable $\mathrm{G}(\mathrm{N}, \mathrm{p}, \alpha)$ with index σ if

$$\sum_{n=1}^{\infty} n^{\sigma-1} \left| N_n^{(\sigma)} - N_{n-1}^{(\sigma)} \right| < \infty \qquad (1.1.6)$$

where $N_n^{(\sigma)} \equiv (N_n^{\epsilon,\alpha})^{(\sigma)}$.

Definition 2:

If $N_n^{\epsilon,\alpha}(s) = 0(1)$, we say that the sequence $\{s_n\}$ is bounded $G(N, p, \alpha)$ where $N_n^{\epsilon,\alpha}(s)$ is defined by (1.1.4).

Definition 3:

Strong summability: A series $\sum_0^{\infty} a_n$ is said to be strongly summable $\mathrm{G}(\mathrm{N}, \mathrm{p}, \alpha)$ with index σ to a if

$$\frac{1}{(\epsilon * \alpha)_n} \sum_{v=0}^{n} |(\epsilon * \Delta\alpha)_v| |T_v^{\epsilon,\alpha}(s) - a|^{\sigma} = 0 \qquad (1.1.7)$$

we denote it by, $s_n \to aG[N, p, \alpha]_{\sigma}$

We remark that this definition is of use only when

$$|(\epsilon * \alpha)_n| \to \infty \quad \text{an } n \to \infty$$

for otherwise, since the expression on the left hand side of (1.1.6) is non-decreasing, (1.1.7) can be satisfied only if, $(\in * \Delta(\alpha s))_v - a(\in * \Delta\alpha)_v = 0$ for every $v \geq 0$ that is, if and only if

$$\alpha_v(s_v - a) = 0$$

for every $v \geq 0$.

We note that, by using (1.1.3),(1.1.5) can be rewritten as

$$T_n^{\epsilon,\alpha}(s) = \frac{(\Delta\epsilon * \alpha s)_n}{(\Delta\epsilon * \alpha)_\mathrm{n}}$$

This shows that $T_n(s)$ is the $G(N, \Delta p, \alpha)$ transform of sequence $\{s_n\}$.

Definition 4:

We say that $\{s_n\}$ is strongly bounded $G(N, p, \alpha)$ with index $\sigma > 0$ or bounded $G(N, p, \alpha)$ if,

$$\frac{1}{(\epsilon * \alpha)_n} \sum_{v=0}^{n} |(\epsilon * \Delta\alpha), \| \mathrm{T}_n^{\epsilon,n}(\mathrm{s})|^\sigma = 0(1). \qquad (1.1.8)$$

where $\mathrm{T}_n^{\epsilon,\alpha}(\mathrm{s})$ defined (1.1.5).

Throughout this paper, for given sequence $\epsilon, \varepsilon, \alpha, \beta, s$ and t we write

$r_n = (\epsilon^* \epsilon)_n$

$\gamma_n = (\alpha * \beta)_n \neq 0$ for all n

$$c_n = \frac{1}{\gamma_n}\sum_{v=0}^{n} \alpha_v s_v \beta_{n,-v} t_{n-v} = \frac{(\alpha s * \beta t)_n}{(\alpha * \beta)_n} \qquad (1.1.9)$$

$$d_n = \frac{1}{\gamma_n}\sum_{v=0}^{n} \alpha_{n-v}\beta_v t_v = \frac{(\alpha * \beta t)_n}{(\alpha * \beta)} \qquad (1.1.10)$$

Also K will denote a positive constant which may not be the same at each occurrence

1.2 INTRODUCTION:

BORWEIN and CASS [2] have established some multiplication theorems concerning strong Nörlund summability and Cauchy product of two series. BORWEIN [I] has established some multiplication theorem involving generalised Nörlund summability and products more general than the Cauchy product of two sequence.

Later on KUMAR [3] give some theorem on strong Nörlund summability and products more general than the Cauchy products of sequences.

Our object in this paper is to generalised all the theorems of KUMAR [3] for $G(N, p, \alpha)$ summability. However our theorems are as follows.

1.3 THEOREMS:

Theorem 1: Suppose that $(\Delta\epsilon * \alpha)_n > 0 (\epsilon * \beta)_n > 0$ for all $n_, (N, \epsilon * \beta, \epsilon * \alpha)$ is regular and $\lambda \geq 1$. If $s_n \to OG[N, p, \alpha]_\lambda$ and $\{t_n\}$ is bounded $G(N, q, \beta)$ then $c_n \to OG(N, r, \gamma)_\lambda$. where c_n is defined by (1.1.9).

Theorem 2: Suppose that $(\Delta\epsilon * \alpha)_n > 0, (\epsilon * \beta)_n > 0$ for all n, $(N, \epsilon * \beta, \epsilon * \alpha)$ is bi-regular and $\lambda \geq 1$. If $s_n \to lG|N, p, \alpha|_\lambda$ and $t_n \to mG(N, q, \beta)$ then $c_n \to lmG(N, r, \gamma)$.

Theorem 3: Suppose that $(\epsilon * \Delta\alpha)_n > 0, (\epsilon * \Delta\beta)_n > 0$ for all n, $\lambda \geq 1$and $(\epsilon * \alpha)_n (\epsilon * \beta)_n =$

$0((\Delta r * \gamma)_n)_{1.2.1}$. If $s_n \to OG[N,p,\alpha]_\lambda$ and t_n is bounded $G[N,q,\beta]_\lambda$ then $c_n \to 0G(N,\Delta r,\gamma)$.

Theorem 4: Suppose that $(\epsilon * \Delta\alpha)_n > 0, (\epsilon * \Delta\beta)_a > 0$ for all $n, \lambda \geq 1$ and (1.2.1) holds. If

$s_n \to lG[N,p,\alpha]_\lambda$ and $t_n \to mG[N,q,\beta]_\lambda$ then $c \to lmG(N,\Delta r,\gamma)$.

Theorem 5: Suppose that $(\epsilon * \Delta\alpha)_n > 0$ for $n \geq 0, \beta_0 > 0$ $\beta_n \geq 0$ for $n > 0, \lambda \geq 1$ and $(N,\beta,\epsilon * \alpha)$ is regular. If $s_n \to OG[N,p,\alpha]_\lambda$, and $\{t_n\}$ is bounded, then $c_n \to OG[N,p,\gamma]_\lambda$.

Theorem 6: Suppose that $(\epsilon * \Delta\alpha)_n > 0$ for $n \geq 0$, $\beta_0 > 0\beta_n \geq 0$ for $n > 0, \lambda \geq 1$ and $(N,\beta,\epsilon * \alpha)$ is bi-regular. If $s_n \to 0G[N,p,\alpha]_\lambda$ and $t_n \to m$ as $n \to \infty$ then $c_n \to lmG[N,p,\gamma]_\lambda$.

For the proof of theorems we required following lemma:

1.4 LEMMA:

If $(\epsilon^*\Delta\alpha)_{n,}, (\epsilon^*\alpha)_n$ be non-zero for all n and

$$\sum_{v=0}^{n} |(\epsilon * \Delta\alpha)_v| = 0((\epsilon * \alpha)_n) \tag{1.4.1}$$

then, for $\lambda > \mu > 0$, $s_n \to lG[N,p,\alpha]_\lambda$ implies $s_n \to lG[N,p,\alpha]_\mu$.

The result of the lemma also hold when summability is replaced by boundedness.

Proof of lemma: Suppose $s_n \to lG[N,p,\alpha]_\lambda$. Thus we have,

$$\frac{1}{(\epsilon^*\alpha)_n}\sum_{v=0}^{n} |(\epsilon * \Delta\alpha)_v||T_v^{\epsilon,\alpha}(s) - \alpha|^\lambda = 0(1) \tag{1.4.2}$$

Using Holder's inequility, we have

$$\frac{1}{(\epsilon * \alpha)_n} \sum_{v=0}^{n} |(\epsilon * \Delta\alpha)_v||T_v^{\epsilon,\alpha}(s) - a|^{\mu}$$

$$\leq \left\{\frac{1}{(\epsilon * \alpha)_n} \sum_{v=0}^{n} |(\epsilon * \Delta\alpha)_v||T_v^{\epsilon,\alpha}(s) - a|^{\lambda}\right\}^{\lambda} \times$$

$$\times \left\{\frac{1}{(\epsilon * \alpha)_n} \sum_{v=0}^{n} |(\epsilon * \Delta\alpha)_n|\right\}^{|\alpha-\mu|)/\lambda} = 0(1)$$

by (1.4.1) and (1.4.2). Hence $s_n \to l\ [N, p, \alpha]_{\mu}$ as desired.

1.5 PROOF OF THEOREMS:

Proof of Theorem 1:

$$(\Delta r * \gamma)_n T_n^{r,\gamma}(c) = (\Delta r * \gamma c)_n$$

$$= (\Delta\epsilon * \alpha s * \beta t)_n$$

$$= \{(\Delta\epsilon * \alpha s)^*(\epsilon * \beta t\}_n$$

$$= \left\{(\Delta\epsilon * \alpha)T^{\epsilon,a}(s)^*(\epsilon * \beta)N^{\epsilon,\beta}(t)\right\}_n$$

Since $\left\{N^{\epsilon,\beta}(t)\right\}$ is bounded, it follows that, for some constant K.

$$\left|(\Delta r * \gamma)_n T_n^{r,\gamma}(c)\right| \leq K\{(\Delta\epsilon * \alpha)|T^{\epsilon,\alpha}(s)| * (\epsilon * \beta)\}_n$$

Hence, using Hölder's inequility, we obtain:

$$\left|(\Delta r * \gamma)_n T_n^{r,\gamma}(c)\right|^{\lambda} \leq K^{\lambda}[\{(\Delta\epsilon * \alpha) * \epsilon * \beta\}_n]^{\lambda-1} \times$$

$$\times \left\{(\Delta\epsilon * \alpha)|T^{\epsilon,a}(s)|^{\lambda}(\epsilon * \beta)\right\}_n$$

$$= K^{\lambda}[(\Delta r * \gamma)_n]^{\lambda-1}\left\{(\Delta\epsilon * \alpha)|T^{\epsilon,\alpha}(s)|^{\lambda} * (\epsilon * \beta)\right\}_n$$

Since$(\Delta\epsilon * \alpha)_n > 0and(\epsilon * \beta)_n > 0$imply that

$$(\Delta r * \gamma)_n > 0, \text{ thus,}$$

$$\sum_{v=0}^{n} (\Delta r * \gamma)_v \left|T_v^{r,\gamma}(c)\right|^{\lambda} \leq K^{\lambda}\left(1 * (\Delta\epsilon^{*}\alpha)|T^{\varepsilon,\alpha}(s)|^{\lambda} * (\epsilon * \beta)\right)_n$$

$$= K^{\lambda} \sum_{v=0}^{n} (\epsilon * \beta)_{n-v} \sum_{\mu=0}^{n} (\Delta\epsilon * \alpha)_{\mu} \left|T_{\mu}^{\epsilon,\alpha}(s)\right|^{\lambda} \qquad (1.5.1)$$

We are given that the inner sum on the right of (1.5.1) is $O((\epsilon * \alpha)_v)$; hence, by the regularity of $(N, \epsilon * \beta, \epsilon * \alpha)$, the expression on right of (1.5.1) is $0((r * \gamma)_n)$. Hence the result.

Proof of Theorem 2:

We write $s_n = l + (s_{n^-} - l)$. The contribution of the second term may be delete with by Theorem 1, so it is enough to consider the case in which s_n is a constant, which may be taken as 1. In other words, we have to prove that if $t_n \to mG(N, q, \beta)$ then $d_n \to m \quad G(N, r, \gamma)_{\lambda}$ where d_n is defined by (1.1.10). Write $t_n = m + (t_n - m)$ we may again consider the case in which $m = 0$.

Now

$$(\Delta r * \gamma)_n T_n^{r,\gamma}(d) = (\Delta r * \gamma d)_n$$

$$= (\Delta r * \alpha * \beta t)_n$$

$$= \left(\Delta \in \alpha * (\epsilon * \beta) N^{\epsilon,\beta}(t)\right)_n$$

Using Hölder's inequility, we find that:

$$\left|(\Delta r * \gamma)_n T_n^{r,\gamma}(d)\right|^{\lambda} \leq \{\Delta\epsilon * \alpha * \epsilon * \beta)_n\}^{\lambda-1} \times$$

$$\times \left\{ \left((\Delta\epsilon * \alpha *) * (\epsilon^{*}\beta) N^{\epsilon,\beta}(t) \right|^{\lambda} \right)_n \right\}$$

$$= \{(\Delta r * \gamma)_n\}^{\lambda-1} \left(\left((\Delta\epsilon * \alpha) * (\epsilon * \beta) \left| N^{\epsilon,\beta}(t) \right|^{\lambda} \right)_n \right)$$

Since,

$(\Delta\epsilon * \alpha), > 0, (\epsilon * \beta)_n > 0$ implies that $(\Delta r * \gamma)_n > 0$ therefore,

$$\sum_{v=0}^{n} (\Delta r * \gamma)_v \cdot \left| T_v^{r,\gamma}(d) \right|^2 \leq \left(1 * (\Delta\epsilon * \alpha) * (\epsilon * \beta) \left| N_{(t)}^{\epsilon,\beta} \right|^{\lambda} \right)_n ,$$

$$= \left(\epsilon * \alpha * (\epsilon, \alpha * \beta) \left| N^{\epsilon,\beta}(t) \right|^{\lambda} \right)_n$$

$$= \sum_{v=0}^{\infty} (\epsilon * \beta), \left| N_v^{\epsilon,\beta}(t) \right|^{\lambda} (\epsilon * \alpha)_{n-v}$$

$$= 0((r * \gamma)_n)$$

Since $\left| N_v^{\epsilon,\beta}(t) \right|^{\lambda} \to 0$ as $n \to \infty$ and $(N, \epsilon * \alpha, \epsilon * \beta)$ isregular.

The proof is thus complete.

Remark:

We remark that the hypothesis of Theorem 3 and 4 imply that $(\Delta r * \gamma)_n > 0$ for all n and $(r * \gamma)_n \to \infty$ as $n \to \infty$, it follows that (for any $\lambda > 0$) $G(N, \Delta r, \gamma)$ summability impliesG$[N, r, \gamma]_\lambda$ summability, so that the conclusion of these theorems are stronger than the $G|N, r, \gamma|_\lambda$ summablity of $\{c_n\}$

Proof of theorem 3:

By the lemma, it is sufficient to prove the theorem for the case $\lambda = 1$ keeping (1.3) in view, we find that

$$N_n^{\Delta r,\gamma}(c) = \frac{(\Delta \mathrm{r} * \gamma c)_n}{(\Delta \mathrm{r} * \gamma)_n}$$

$$= \frac{(r * \Delta(\gamma c))_n}{(r * \Delta\gamma)_n} = T_n^{r,\gamma}(c)$$

Thus we have to prove that $T_n^{r,\gamma}(c) \to 0$ as $\mathrm{n} \to \infty$. Since the hypotheses imply that $(\mathrm{r} * \Delta\gamma)_n >$ 0, therefore,

$$(\mathrm{r} * \Delta\gamma)_n \mathrm{T}_n^{r,\gamma}(\mathrm{c}) = \{|(1 * \epsilon * \epsilon * \Delta(\alpha \mathrm{s}) * \Delta(\beta \mathrm{t}))_n\ |\}$$

where,

$$\theta_n = \frac{1}{(\epsilon * \beta)_0} \sum_{v=1}^{n} (\epsilon * \Delta\beta)_v \left| T_v^{\epsilon,\beta}(t) \right|$$

Since $\{t_n\}$ is bounded $G[N, q, \beta]$, implies that

$\theta_n = 0(1)$,thus we obtain

$$\left|T_n^{r,\gamma}(c)\right| \le \frac{k}{(\Delta r * \gamma)_n} \left\{ \left((\epsilon * \Delta\alpha) |T_n^{\epsilon,\alpha}(s)| * (\epsilon * \beta) \right)_n \right\}$$

$$= \frac{K}{(\Delta r * \gamma)_n} \left\{ \sum_{v=0}^{n} (\epsilon * \Delta\alpha)_v |T_v^{\epsilon,\alpha}(s)| (\epsilon * \beta)_{n-v} \right\}$$

Since $(\epsilon * \Delta\beta)_n > 0$, it is follows that $\{(\epsilon * \beta)_n\}$ is increasing. Also by hypothesis

$$\sum_{v=0}^{n} (\epsilon * \Delta\alpha)_v |T_v^{\epsilon,\alpha}(s)| = 0((\epsilon * \alpha)_n)$$

Hence,

$$|\ T_n^{r,\gamma}(c) \le \frac{K(\epsilon * \beta)_n}{(\Delta r * \gamma)_n} \sum_{v=0}^{n} (\epsilon * \Delta\alpha)_v |T_v^{r,\alpha}(s)|$$

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$$= 0\left(\frac{(\epsilon * \beta)_n (\epsilon * \alpha)_n}{(\Delta r * \gamma)_n}\right) = 0(1)$$

by (1.2.1). The proof is thus complete.

Proof of Theorem 4:

By the Lemma, it is sufficient to prove the theorem for the case $\lambda = 1$.

As in the proof of Theorem 2 we write $s_n = l + (s_n - l)$, the contribution of the second term may be delete with by Theorem 3, so it is enough to consider the case in which s_n is a constant, which may be taken as 1 . In other words, we have to prove that, if $t_n \to mG[N, q, \beta]$, then $d_n \to mG[N, \Delta r, \gamma]$ where d_n is defined by (1.1.10). Writing $t_n = m + (t_n - m)$ we may again consider the case in which $m = 0$. So we have to prove that $T_n^{r,\gamma}(d) \to 0$ as $n \to \infty$

Now

$$(\Delta r * \gamma)_n \left|T_n^{r,\gamma}(d)\right| \leq \left(\epsilon * \alpha * (\varepsilon * \Delta\beta)\left|T^{\epsilon,\beta}(t)\right|\right)_n$$

$$= \sum_{v=0}^{n} (\epsilon * \alpha)_{n-v} (\varepsilon * \Delta\beta)_v \left|T_v^{\epsilon,\beta}(t)\right|$$

since $(\in * \Delta\alpha)_n > 0$, it follows that $\{(\in * \alpha)_n\}$ is increasing

Thus

$$\left|T_n^{r,\gamma}(d)\right| \leq \frac{(\epsilon * \alpha)_*}{(\Delta r * \gamma)_n} \sum_{v=0}^{n} (\varepsilon * \Delta\beta)_v \left|T_v^{\epsilon,\beta}(t)\right|$$

$$= 0\left(\frac{(\epsilon * \alpha)_n (\varepsilon * \beta)_n}{(\Delta r * \gamma)_n^*}\right) = 0(1)$$

Since $t_n \to OG[N, q, \beta]$, and condition (1.2.1) holds. Hence proof is complete

Proof of Theorem 5:

Now

$$\left\{(\in * \Delta\gamma)_n \left|T_n^{\epsilon,\gamma}(c)\right|\right\}^{\lambda} = |(\epsilon * \Delta(\alpha s) * \beta t)_n|^{\lambda}$$

$$\leq \{((\epsilon * \Delta(\alpha)|T^{\epsilon,\alpha}(s)| * \beta|t|)_n\}^{\lambda}$$

$$\leq K\{((\in * \Delta\alpha)|T^{\epsilon,\alpha}(s)| * \beta)_n\}^{\lambda}$$

Since $\{t_n\}$ is bounded using Hölder's inequality, we find that,

$$\left\{(\in * \Delta\gamma)_n \left|\mathrm{T}_n^{\epsilon,\gamma}(\mathrm{c})\right|\right\}^{\lambda} \leq \mathrm{K}\{(\in * \Delta\gamma)_n\}^{\lambda-1}\left\{\left((\in * \Delta\alpha) \times |\mathrm{T}^{\epsilon,\alpha}(\mathrm{s})|^{\lambda} * \beta\right)_n\right\}$$

Hence,

$$\frac{1}{(\epsilon^*\gamma)_n}\sum_{v=0}^{n}(\epsilon * \Delta\gamma)_v\left|T_n^{\epsilon,\gamma}(c)\right|^{\lambda} \leq \frac{k}{(\epsilon * \gamma)}\sum_{n=0}^{n}\beta_v \cdot \sum_{\mu=0}^{n-\gamma}(\epsilon * \Delta\alpha)_{\mu}|T_n^{\epsilon,\alpha}(s)|^{\lambda} = 0(1)$$

Since, by hypotheses

$$\sum_{\mu=0}^{n}(\epsilon * \Delta\alpha)^{\mu}|\mathrm{T}_n^{\epsilon,\alpha}(\mathrm{s})|^{\lambda} = 0((\epsilon * \alpha)_{n-v})$$

and $(N, \beta, \epsilon * \alpha)$ is regular

Thus the proof is complete.

Proof of Theorem 6:

Writing $\mathrm{s_n} = l + (\mathrm{s_n} - l)$. The contribution of the second term may be dealt with by Theorem 5 , so that it is enough to consider the case in which s_n is a constant, which may be taken as 1 . In other words, we have to prove that, if $t_n \to m$ as $\mathrm{n} \to \infty$, then $\mathrm{d_n} \to \mathrm{mG}[\mathrm{N}, \mathrm{p}, \gamma]_{\lambda}$, where d_n is defined by $(1.1.10)$. Writing $t_n = m + (t_n - m)$, we may again consider the case in which m = 0.

Now

$$\left\{(\in * \Delta\gamma)_n \mid T_n^{\epsilon,\gamma}(\mathrm{d})\right\}^\lambda = (\epsilon * \Delta\alpha * \beta \mathrm{t})_n|^\lambda$$

$$\leq \{(\in * \beta|\mathrm{t}| * \Delta\alpha)_n\}^\lambda$$

By Hölder's inequality, we obtain,

$$\left\{(\in * \Delta\gamma)_n \left|\mathrm{T}_\mathrm{n}^{\epsilon,\gamma}(\mathrm{d})\right|\right\}^\lambda \leq \{(\in * \Delta\alpha * \beta)_n\}^{\lambda-1} \times \left\{\left(\in * \Delta\alpha * \beta|\mathrm{t}|^\lambda\right),\right\}$$

Hence

$$\frac{1}{(\epsilon * \gamma)_n} \sum_{v=0}^{n} (\epsilon^* \Delta\gamma), \left|T_v^{\epsilon,\gamma}(\mathrm{d})\right|^\lambda$$

$$\leq \frac{1}{(\epsilon * \gamma)_n} \sum_{v=0}^{n} \beta_v |t_v|^\lambda \left[\sum_{\mu=0}^{n-1} (\epsilon * \Delta\alpha)_\mu\right]$$

$$= \frac{1}{(\epsilon * \gamma)_n} \sum_{v=0}^{n} (\epsilon * \alpha)_{n-v} \beta_v |t_v|^\lambda$$

$$= 0(1)$$

Since $|t_n|^\lambda \to 0$ as $n \to \infty$ and $(N, \epsilon * \alpha, \beta)$ is regular.

This complete the proof.

REFERENCES

[1] BORWEIN, D : On product of sequences. Journ. London Math. Soc 33(212-220) 1958

[2] BORWEIN, D and : Multiplication theorem for strong Nörlund Summability. Math.

CASS, F.P. Z.107, (33-42)(1968)

[3] KUMAR, A : Multiplication theorem for Generalized strong Nörlund summability. Indian J. pure Appl.Math. 14(11)(1388-1397) (1983)